# Loops and their varieties

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Quasigroups form a variety with binary operations $\cdot$ (multiplication), $/$ (left division) and $\backslash$ (right division) such that $(xy)/y = x$, $x\backslash(xy) = y$, $(x/y)y = x$ and $x(x\backslash y) = y$. Loops are quasigroups with a neutral element. That can be expessed by identities $x1 = x = 1x$.

The most prominent subvariety of loops are Moufang loops, for which there can be used any of the identities $x(y \cdot xz) = (xy \cdot x)z$, $x(yz \cdot x) = xy \cdot zx$ and $(zx \cdot y)x = z(x \cdot yx)$. Closely related are the left Bol law $x(y \cdot xz) = (x \cdot yx)z$ and the right Bol law $(zx \cdot y)x = z(xy \cdot x)$. It is usual to say that a multiplicative identity $s(x, y, z) = t(x, y, z)$ is of Bol-Moufang type if the variables in $s$ and $t$ appear in the same order, with $x$ occuring twice and with $y$ and $z$ occuring once each. Quite suprisingly, besides trivialities this general approach does not bring much new: we get a subvariety of Moufang loops that is expressed by the so called extra laws, and we get the laws $(x \cdot xy)z = x^2y \cdot z$, $z \cdot yx^2 = z(yx \cdot x)$ and $y(x \cdot xz) = (yx \cdot x)z$. We shall argue that this somewhat formal approach should be replaced by a process called nuclear identification which yields not only all of the above laws, but also the laws of conjugacy closedness and the Buchsteiner law $x\backslash(xy \cdot z) = (y \cdot zx)/x$.

Nuclear identification is based upon expressing the loop laws by appropriate autotopisms and can be used to explain why various loop laws are equivalent. This amounts, in fact, to a new and more systematic introduction to the loop theory. Time allowing we shall also mention a simplified proof of the Moufang theorem, and compare the traditional definition of soluble loops with the definition supplied by the general commutator theory of universal algebra.